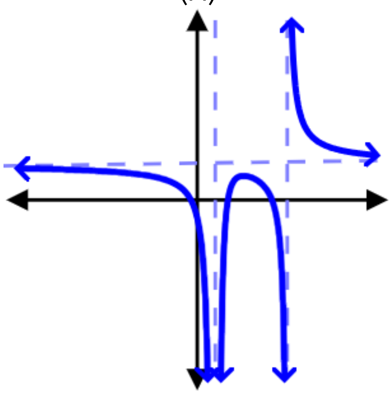
Rational Function End Behavior (1.7) HW

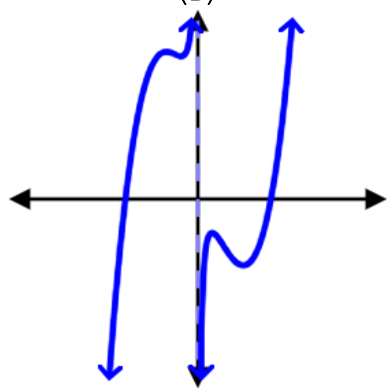
Below are nine rational function equations and nine graphs. Use end behavior to match each equation to the letter of the corresponding graph. Use each graph/equation exactly once. There are no repeats.

1. $y = \frac{2x^3 - 11x^2 + 10x + 5}{x^3 - 7x^2 + 11x - 5}$
2. $y = \frac{3}{x^2 - 10x + 25}$
3. $y = \frac{5x^2 - 31x + 24}{3x - 15}$
4. $y = \frac{-x^2 - x + 8}{2x - 6}$
5. $y = \frac{-x + 1}{x + 1}$
6. $y = \frac{-x^3 + 63x - 164}{8x - 32}$
7. $y = \frac{-x^4 + 21x^2 - 20x - 12}{6x - 6}$
8. $y = \frac{x^4 - x^3 - 16x^2 + 16x - 10}{5x}$
9. $y = \frac{x^3 + 1}{5x - 8}$

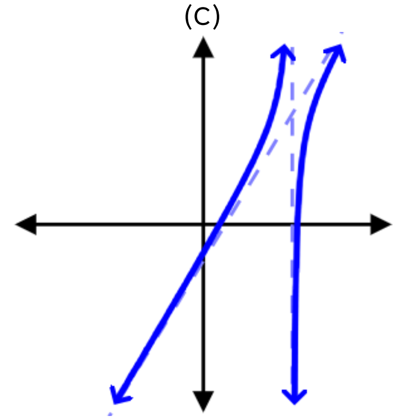
(A)



(B)



(C)



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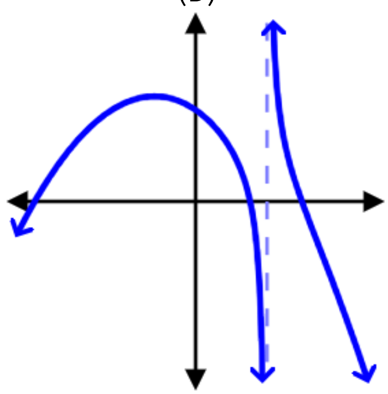
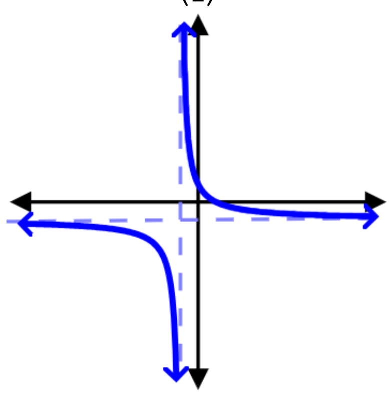


(E)



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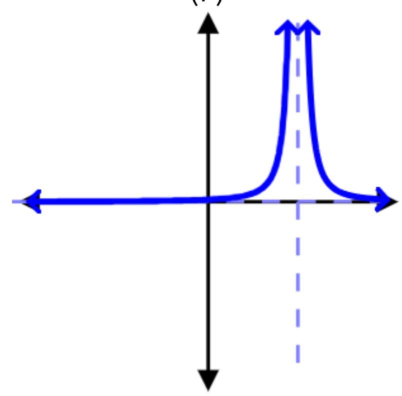
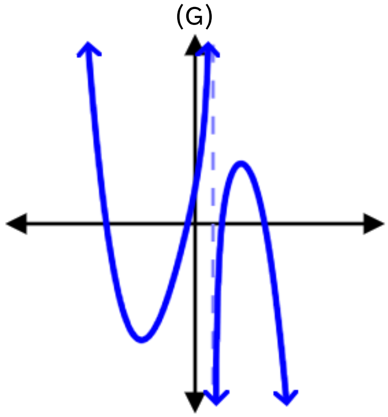
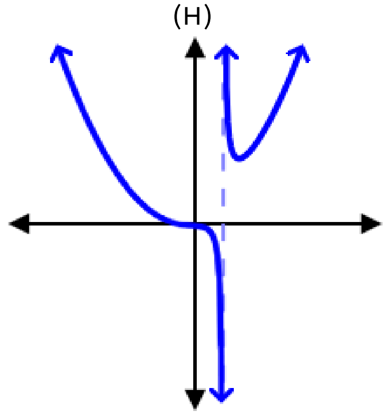


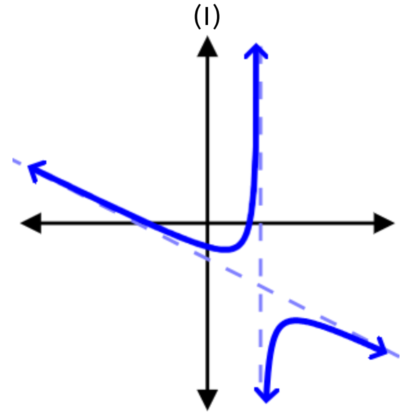
(G)



(H)



(I)



For each of the following functions, determine $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

10. $f(x) = \frac{-2x^2+6x-5}{3x+1}$

11. $f(x) = \frac{3x+1}{-2x^2+6x-5}$

12. $f(x) = \frac{(x^2-9)^2}{x-6}$

13. $f(x) = \frac{x-6}{(x^2-9)^2}$

14. $f(x) = \frac{(x+3)(2x-1)^2}{(x^2-9)(9x+1)}$

15. $f(x) = \frac{3x-2x^2+5}{7x^2+10-2x^3}$

For each of the following functions, determine the horizontal asymptote. For functions that do not have a horizontal asymptote, state "none."

16. $y = \frac{x}{2x^2+5x-3}$

17. $y = \frac{2x^2+5x-3}{x}$

18. $y = \frac{2x^2+5x-3}{4x^2-1}$

19. $y = \frac{2x^2+x^5-3}{4x^2-1}$

Each of the following questions describes a rational function $f(x)$ that is of the form $f(x) = \frac{n(x)}{d(x)}$ where $n(x)$ and $d(x)$ are polynomial functions. Decide if the scenario is possible or impossible. If impossible, give a reason why.

20. $n(x)$ is linear, $5-i$ is a zero of $d(x)$, $\lim_{x\to\infty} f(x) = 4$, $\lim_{x\to-\infty} f(x) = -4$

21. $n(x)$ is of an even degree, $d(x)$ is of an odd degree, $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to\infty} f(x) = \infty$

22. $n(x)$ has three real zeros (not necessarily distinct), $d(x)$ has no real zeros $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$

23. $n(x)$ has three real zeros (not necessarily distinct), $d(x)$ has no real zeros, $\lim_{x\to-\infty} d(x) = -\infty$, $\lim_{x\to\infty} f(x) = 0$

24. $n(x)$ has three real zeros (not necessarily distinct), $d(x)$ has no real zeros, $\lim_{x\to-\infty} d(x) = -\infty$, $\lim_{x\to\infty} f(x) = 2$

Rational Function End Behavior (1.7) HW Answers

1. A
2. F
3. C
4. I
5. E
6. D
7. G
8. B
9. H
10. $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$
11. $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$
12. $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$
13. $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$
14. $\lim_{x\to-\infty} f(x) = \frac{4}{9}$ and $\lim_{x\to\infty} f(x) = \frac{4}{9}$
15. $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$
16. $y = 0$
17. None
18. $y = \frac{1}{2}$
19. None
20. Impossible. A few reasons this is impossible.
    One way to reason this is impossible: The third and fourth statement are contradictory. If $\lim_{x\to\infty} f(x) = 4$, then it must be the case that $\lim_{x\to-\infty} f(x) = 4$ as well. Rational functions can have at most one horizontal asymptote.
    Another reason this is impossible: If $5 - i$ is a zero of $d(x)$, then $5 + i$ must also be a zero of $d(x)$. Since $d(x)$ has at least two zeros, $d(x)$ is of degree at least two. Since $n(x)$ is only linear (degree 1), we know $d(x)$ is of a greater degree than $n(x)$. When the denominator of a rational function is of a higher degree than the numerator, the limit at either end of the function must be zero. $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$ based on the degrees of the numerator and denominator. This contradicts the third and fourth statement
21. Impossible. If $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = \infty$, then the leading terms of $n(x)$ and $d(x)$ must have divided to make a polynomial with an even degree. But if $n(x)$ is of an even degree and $d(x)$ is of an odd degree, it's impossible they divided to yield an even degree.
22. Possible. In fact, $f(x) = \frac{x^3}{x^2+1}$ satisfies all of the given statements.
23. Possible. In fact, $f(x) = \frac{x^3}{-(x^4+1)}$ satisfies all of the given statements.
24. Impossible. $n(x)$ has three real zeros. We don't know how many non-real zeros $n(x)$ has, but we do know they must come in pairs, so we do know that overall $n(x)$ has an odd number of zeros. $n(x)$ has an odd degree. Since $\lim_{x\to\infty} f(x)$ equals a non-zero finite number, we know that $n(x)$ and $d(x)$ have the same degree. So $d(x)$ is of an odd degree. But all odd degree polynomials must have at least one real zero. This contradicts the second statement.